Probability and Random Processes

ECS 315

Asst. Prof. Dr. Prapun Suksompong

prapun@siit.tu.ac.th

8 Discrete Random Variable

Office Hours:

Check Google Calendar on the course website.

Dr.Prapun's Office:

6th floor of Sirindhralai building, BKD

Discrete Random Variable

- A random variable is **discrete** if its values can be limited to only a **countable** number of possibilities.
- Recall that "countable" means
  - finite or
  - Countably infinite.
- Crucial skill 8.1.1: Determine whether a RV is discrete.

HW 7

Problem 1. For each description of a random variable X below, indicate whether X is a **discrete** random variable.

(a) X is the number of websites visited by a randomly chosen software engineer in a day.

(b) X is the number of classes a randomly chosen student is taking.

(c) X is the average height of the passengers on a randomly chosen bus.

(d) A game involves a circular spinner with eight sections labeled with numbers. X is the amount of time the spinner spins before coming to a rest.

(e) X is the thickness of the longest book in a randomly chosen library.

(f) X is the number of keys on a randomly chosen keyboard.

(g) X is the length of a randomly chosen person's arm.

Chapter 5 vs. Chapter 8

- In Chapter 5, probability of any countable event can be found by knowing the probability $P(\{\omega\})$ for each outcome ω.
- In Chapter 8, probability of any statement about a discrete RV X can be found by using probability of the form $P[X = x]$ (without referring back to the outcomes and the sample space).
  - Because $P[X = x]$ is important and use frequently, as a function of x, we name it the **probability mass function (pmf)**.
  - Definition: $p_X(x) \equiv P[X = x]$

Section 8.1

- Crucial skill 8.1.1: Determine whether a RV is discrete.
- Crucial skill 8.1.2: Determine the probability mass function (pmf) of a discrete RV when it is defined as a function of outcomes (as in Chapter 7).

$$p_X(x) \equiv P[X = x]$$

Chapter 7 vs. Chapter 8

- In Chapter 7, RV are defined as a function of the outcomes.
- In Chapter 8, we want to talk about RV directly, skipping the outcomes.
  - So, need to find ways to calculate probability without going back to the sample space.

Chapter 5:
Probability of any event can be found by knowing the probability $P(\{\omega\})$ for each outcome ω.

Chapter 7:
Probability of any statement about a RV can be found by converting the statement back into a collection of outcomes satisfying the statement.
- Still use $P(\{\omega\})$

Chapter 8:
- $P(\{\omega\})$ is not available.

Probability of any statement about a discrete RV X will be found by using probability of the form $P[X = x]$.

Example 8.16: pmf and probabilities

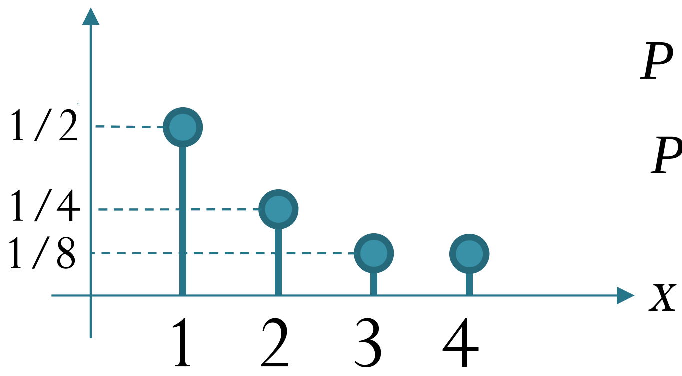
Consider a random variable (RV) X.

probability mass function (pmf)

$$p_X(x) = P[X = x]$$

$$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3, 4\} \\ 0, & \text{otherwise} \end{cases}$$

stem plot:



$$P[X = 2] = ?$$

$$P[X > 1] = ?$$

Example 8.16: pmf and probabilities

Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

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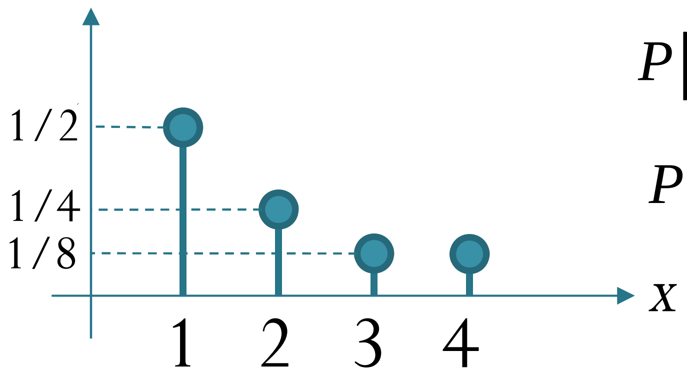


$$P[X = 2] = p_X(2) = \frac{1}{4}$$

$$P[X > 1] = p_X(2) + p_X(3) + p_X(4)$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$$

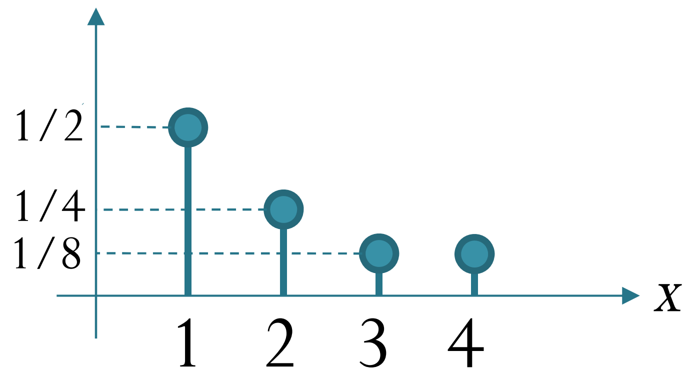
Example: pmf and its interpretation

Consider a random variable (RV) X.

probability mass function (pmf) ?

$$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

stem plot:



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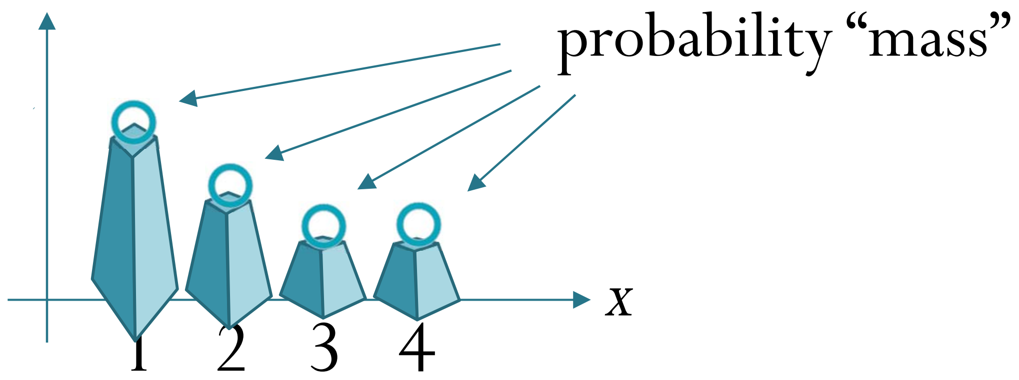

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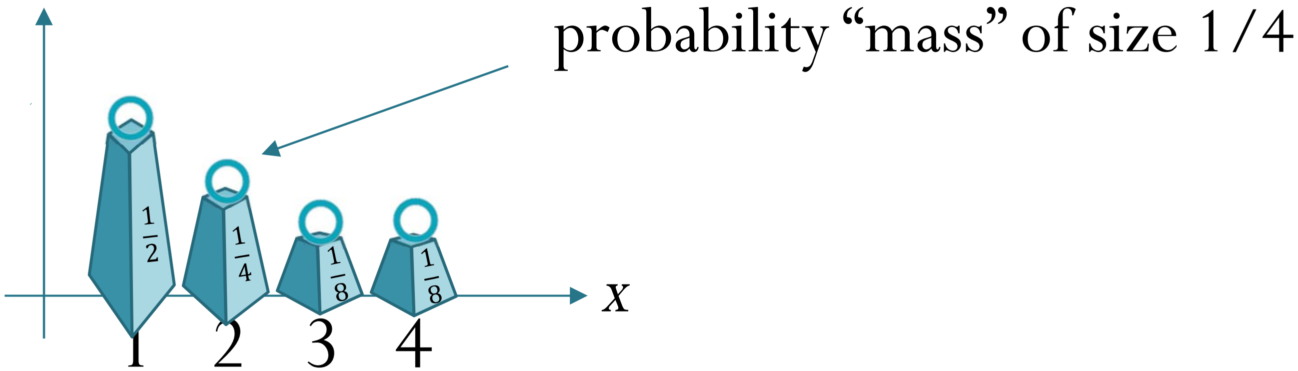
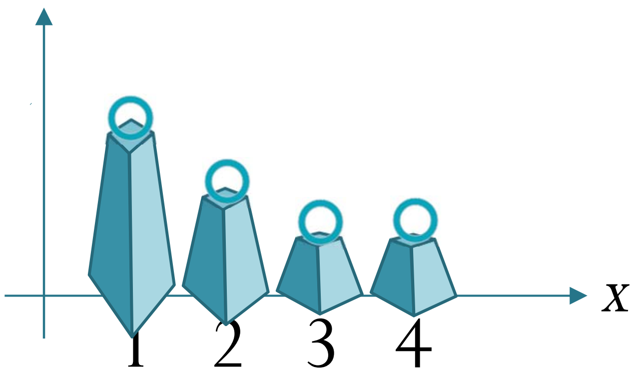


Example: Support of a RV

Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$



What about the **support** of this RV X?

Example: Support of a RV

Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

The set $\{1,2,3,4\}$ is a support of X.

Example: Support of a RV

Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

The set $\{1,2,2.5,3,4,5\}$ is also a support of this RV X.

Example: Support of a RV

Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

The set $\{1,2,4\}$ is *not* a support of this RV X.

Example: Support of a RV

Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

The set $\{1,2,3,4\}$ is the "minimal" support of X.

For discrete RV, we take the collection of x values at which $p_X(x) > 0$ to be our **"default" support**.

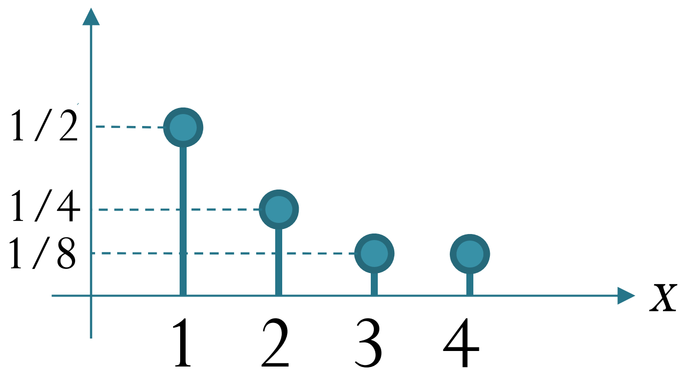
Example: Support of a RV

Consider a random variable (RV) X.

probability mass function (pmf) $p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$

$p_X(x) = P[X = x]$

stem plot:



The "default" support for this RV is the set $S_X = \{1,2,3,4\}$.

Section 8.1

- Crucial skill 8.1.1: Determine whether a RV is discrete.
- Crucial skill 8.1.2: Determine the probability mass function (pmf) of a discrete RV when it is defined as a function of outcomes (as in Chapter 7).
- **Crucial skill 8.1.3**: Given the pmf of a discrete RV,
  - find the value of an unknown constant in the pmf,
  - sketch the pmf
    - always use stem plot
  - calculate probability of a statement about the RV
  - find and plot the cdf

Example: CDF

Back to Example 8.16

Consider a random variable (RV) X.

probability mass function (pmf)

$$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3, 4\} \\ 0, & \text{otherwise} \end{cases}$$

$$p_X(x) = \text{P}[X = x]$$

cumulative distribution function (cdf)

$$F_X(x) = \text{P}[X \le x]$$

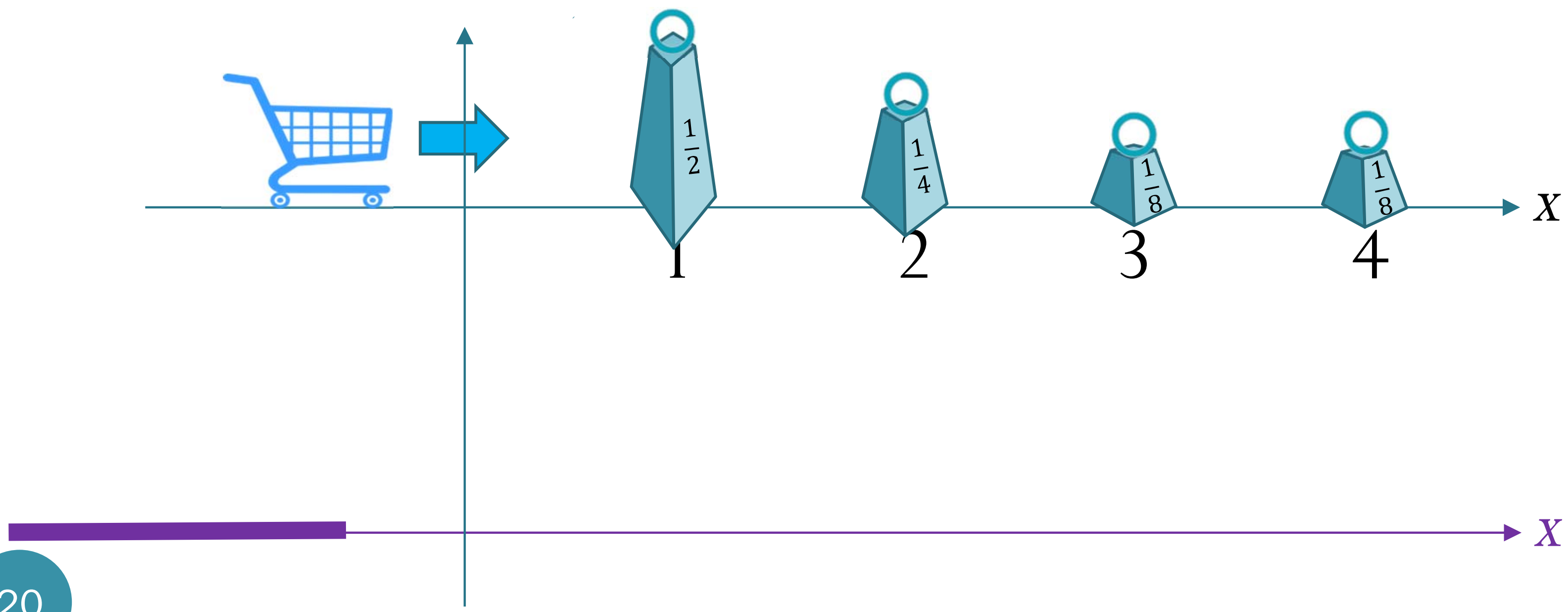
Example: CDF

Back to Example 8.16

Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3, 4\} \\ 0, & \text{otherwise} \end{cases}$$

cumulative distribution function (cdf)



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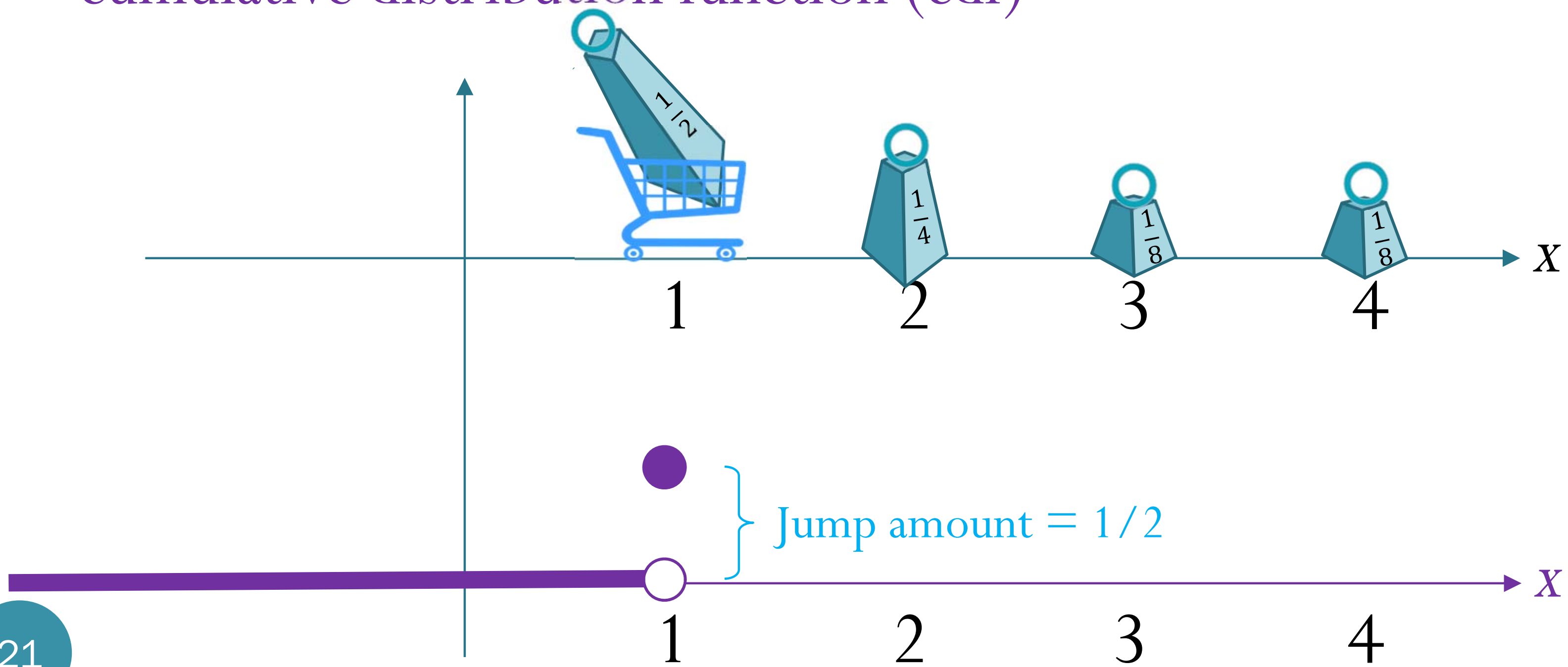

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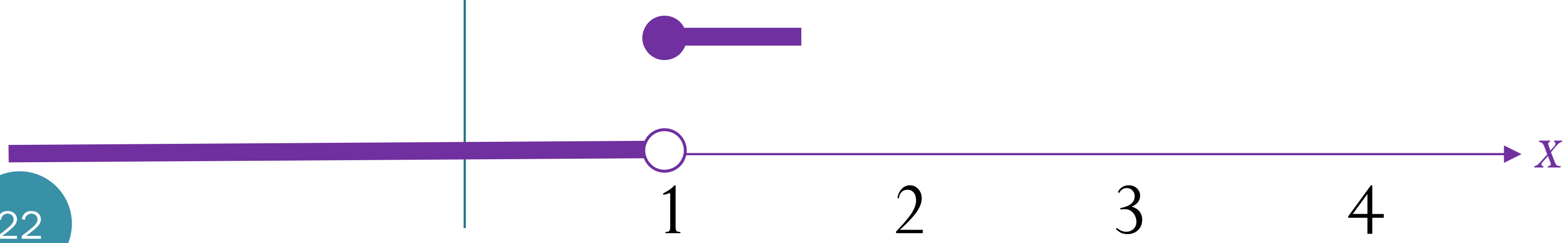

Example: CDF

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Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3, 4\} \\ 0, & \text{otherwise} \end{cases}$$

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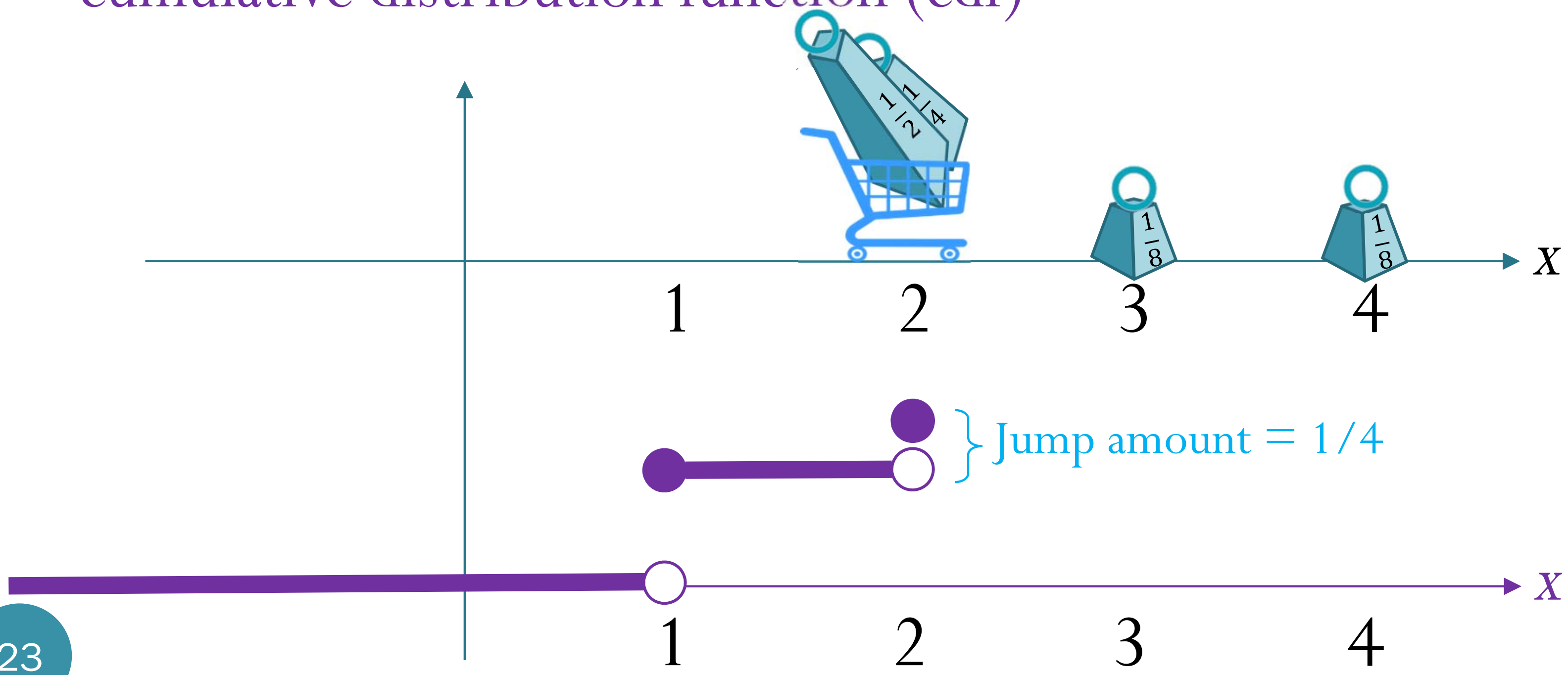

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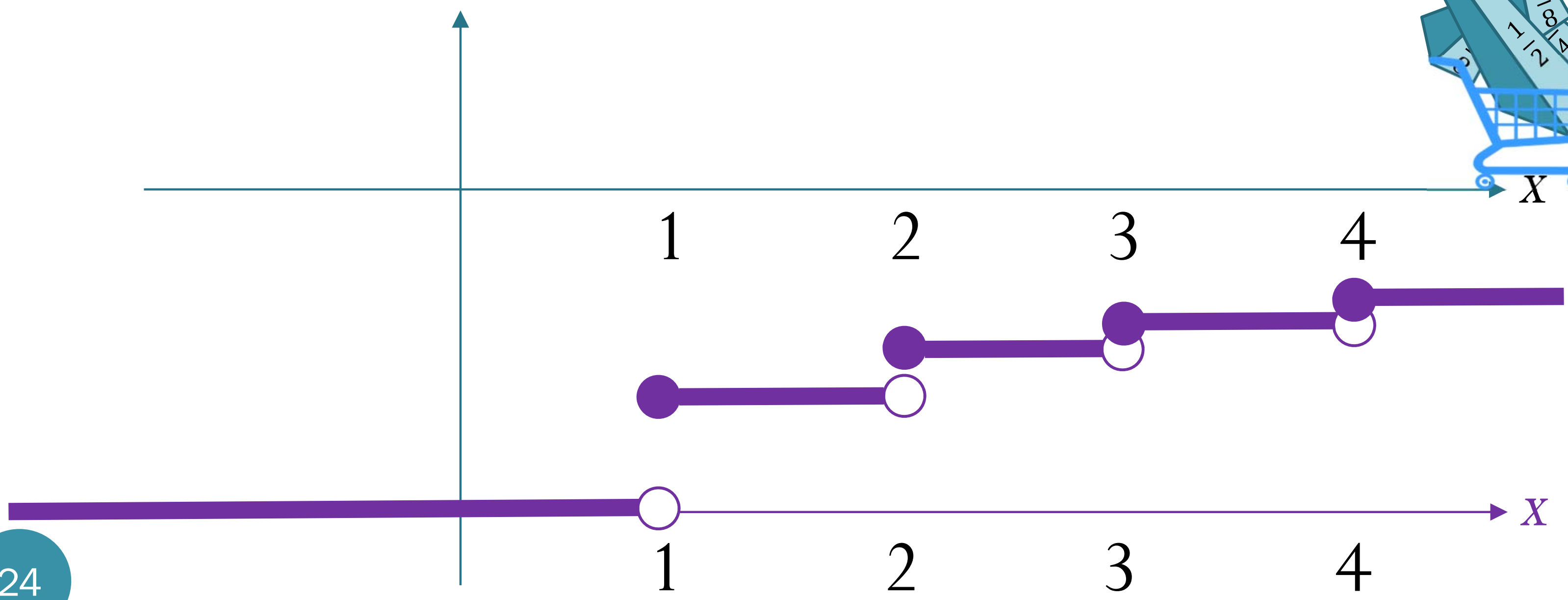

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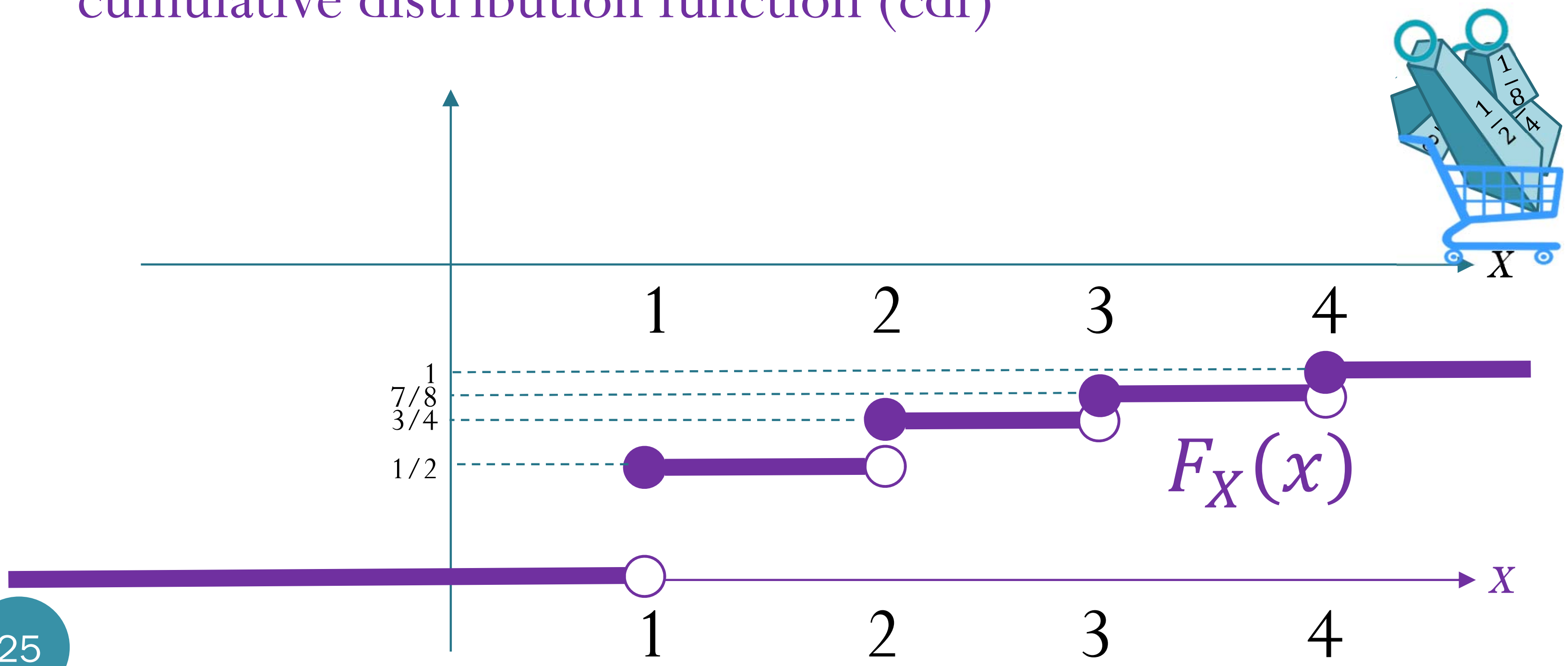

Example: CDF

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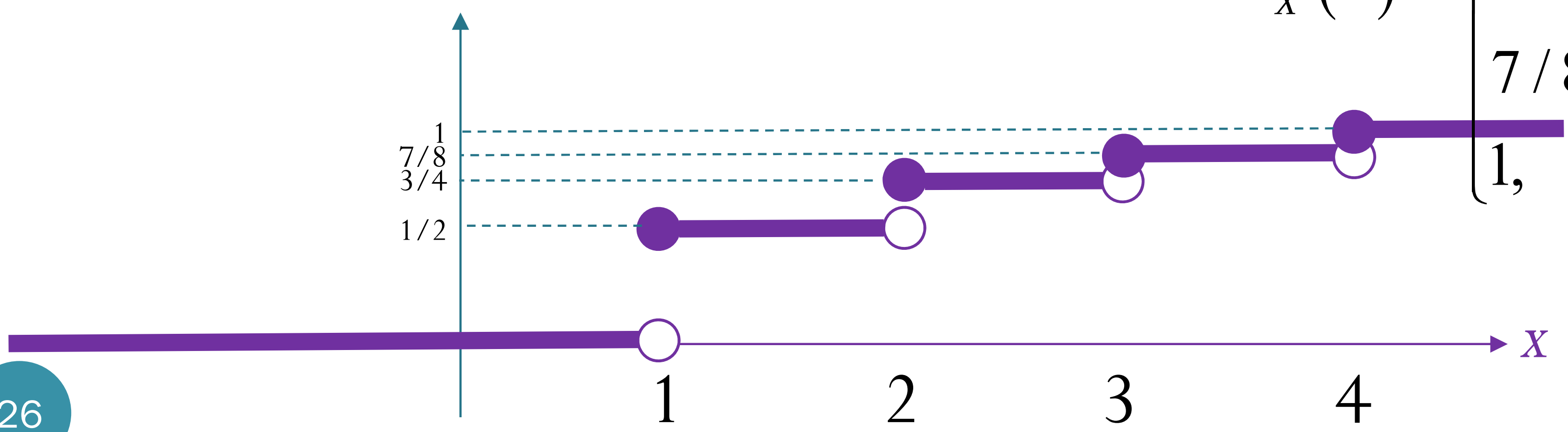
Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

cumulative distribution function (cdf)

$$F_X(x) = \begin{cases} 0, & x < 1, \\ 1/2, & 1 \le x < 2, \\ 3/4, & 2 \le x < 3, \\ 7/8, & 3 \le x < 4, \\ 1, & x \ge 4. \end{cases}$$



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- **Crucial skill 8.1.3**: Given the pmf of a discrete RV,
  - find the value of an unknown constant in the pmf,
  - sketch the pmf
    - always use stem plot
  - calculate probability of a statement about the RV
  - find and plot the cdf
- **Crucial skill 8.1.4**: Given the cdf of a discrete RV,
  - calculate probability of a statement about the RV
  - find and plot the pmf

Example: CDF

Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3, 4\} \\ 0, & \text{otherwise} \end{cases}$$

cumulative distribution function (cdf)